Class 12 Mathematics – Chapter: Inverse Trigonometric Functions

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1. Introduction

- Inverse trigonometric functions reverse the effect of trigonometric functions.
- They help find angles when the value of the function is known.

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2. Definition

- If $y = \sin x$, then $x = \sin^{-1} y$ (or $\arcsin y$) is the inverse function of sine.
- Similarly for cosine, tangent, cotangent, secant, and cosecant.

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3. Principal Values and Domains

- To have inverse functions, restrict the domain of original functions:
  - $\sin^{-1} x: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
  - $\cos^{-1} x: [-1, 1] \to [0, \pi]$

- $\tan^{-1} x: \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\cot^{-1} x: \mathbb{R} \to (0, \pi)$
- $\sec^{-1} x: (-\infty, -1] \cup [1, \infty) \to \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
- $\csc^{-1} x: (-\infty, -1] \cup [1, \infty) \to \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

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4. Basic Properties

- $\sin(\sin^{-1} x) = x$, for $x \in [-1,1]$
- $\cos(\cos^{-1} x) = x$, for $x \in [-1,1]$
- $\tan(\tan^{-1} x) = x$, for all real x

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5. Important Identities

- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
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- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\sec^{-1} x = \cos^{-1} \frac{1}{x}$
- $\csc^{-1} x = \sin^{-1} \frac{1}{x}$

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6. Derivatives of Inverse Trigonometric Functions

- $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
- $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$
- $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

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7. Applications

- Solving equations involving trigonometric functions.

- Calculus problems involving integration and differentiation.
- Geometry and physics problems involving angles.

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8. Exam Tips

- Memorize principal domains and ranges.
- Practice using identities to simplify expressions.
- Understand derivative formulas well.
- Solve problems on inverse functions and their applications.